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Original Contribution

MEASURABILITY WITH RESPECT TO GROUP OF MOTIONS IN THE GALILEAN SPACE G3

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ABSTRACT

The paper deals with measurability of sets of geometrics elements with respect to the group of the Galilean motions in the Galilean space G_3.

Keywords: measurable, group of the motions in the Galilean space G_3.

INTRODUCTION

The geometry of the Galilean space G_3 has been largely developed by Otto R)schel in [5]

The space G_3 is defined (see [5]) as a projective space **P** (R) in which the absolute consists of a real plane w (the absolute plane) and a real line $f \in$ w (the absolute line) with an elliptic involution J defined on it$_1$.

In homogenous coordinates $(x_0 : x_1 : x_2 : x_3) \neq (0:0:0:0)$ we take as usually the plane $x_0 = 0$ as the plane w, the line $x_0 = x_1 = 0$ as the line f and $J : (0:0:x_2:x_3) \to (0:0:x_3:-x_2)$ as the involution J.

The six-member group of transformations

B′$_6$

$$\begin{cases} x_0 = \bar{x}_0 \\ x_1 = c_1\bar{x}_0 + \bar{x}_1 \\ x_2 = c_2\bar{x}_0 + c_3\bar{x}_1 + \cos\varphi\,\bar{x}_2 + \sin\varphi\,\bar{x}_3 \\ x_3 = c_4\bar{x}_0 + c_5\bar{x}_1 - \sin\varphi\,\bar{x}_2 + \cos\varphi\,\bar{x}_3 \end{cases}$$

is called the group of the Galilean motions in G_3 [4].

The six-parameters group B′$_6$ of transformations, in affine coordinates, has the form B_6

$$\begin{cases} x = c_1 + \bar{x} \\ y = c_2 + c_3\bar{x} + \bar{y}\cos\varphi + \bar{z}\sin\varphi \\ z = c_4 + c_5\bar{x} - \bar{y}\sin\varphi + \bar{z}\cos\varphi \end{cases} \quad (1)$$

$c_1, c_2, c_3, c_4, c_5, f \in$ **R.**

The aim of this paper is to present some integral geometric results, under B_6 in the Galilean space G_3.

Theorem (R. Deltheil).
The function $f(x_1,x_2,\ldots x_n)$ is an integral invariant function of the group $G_r(a)$ when it satisfies the system

$$X_k(f) + \sigma_k f = 0, \qquad k = 1, 2, \ldots, r\,.$$

Where $\sigma_k = \sum_{i=1}^{n} \frac{\partial \xi_i^k}{\partial x_i}$.

The differential form

$$dx = \left| f(x_1,\ldots,x_n) \right| dx_1 \wedge \ldots \wedge dx_n$$

is called an invariant density under the group $G_r(a)$ of the elements $x(x_1, \ldots, x_n)$.

Let M_q be q-parametric set of p-dimensional geometrical elements determined by the system $\varphi_j(x_1,\ldots,x_n, \alpha_1,\ldots,\alpha_q) = 0$, j = 1, 2, …, n-p,

where $\alpha_1,\ldots,\alpha_q$ are independent parameters. If the group $G_r(a)$ leaves M_q invariant , then it generates in the set of parameters $E_q(\alpha)$, so-called associated group $\overline{G}_r(\alpha)$, [8; p.33]. The group $\overline{G}_r(\alpha)$ is isotropic to the group $G_r(a)$, and can be determinate by a system

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$\alpha'_j = \psi_j(\alpha_1,...,\alpha_q; a_1,...,a_r)$, j = 1, 2, …, q.

1. Density for sets of points

Let P(x, y, z) be an arbitrary points in G_3 . The group B_6 acts transitively on each set of points M = { P (x, y, z) } and the unique 3-form on M, invariant under the action of (1) is

$$dP = dx \wedge dy \wedge dz. \quad (2)$$

The invariant density with respect to B_6 for the points P (x, y, z) in G_3 is (2).

2. Density for sets of planes

The plane in G_3 with equation π: A x + B y +C z +D = 0 is said to be *isotropic* if $B^2 + C^2 \neq 0$. The plane with equation x = A is said to be *Euclidean*. By the action of B_6 an Euclideanplane is transformed into an Euclidean plane. Let π: Ax+By +Cz +D = 0 be an isotropic plane, and let $C \neq 0$. Then

$$\pi:\ z = -\frac{A}{C}x - \frac{B}{C}y - \frac{D}{C}.$$

If $u = -\frac{A}{C}$, $v = -\frac{B}{C}$, $w = \frac{-D}{C}$, then π*: z = ux + vy +w*. Under the action of the transformations (1), The plane π(*u, v, w)* is transformed into the plane $\pi(\bar{u},\bar{v},\bar{w})$, where

$$\bar{u} = \frac{u + (c_3 + vc_5)\sin\varphi + (c_5 - vc_3)\cos\varphi}{\cos\varphi + v\sin\varphi} \quad (3)$$

$$\bar{v} = \frac{v\cos\varphi - \sin\varphi}{\cos\varphi + v\sin\varphi}$$

$$\bar{w} = \frac{w - uc_1 + (vc_4 - vc_1c_5 - c_1c_3 + c_2)\sin\varphi + (vc_1c_3 - vc_2 - c_1c_5 + c_4)\cos\varphi}{\cos\varphi + v\sin\varphi}$$

The transformations (3), form the so-called associated group $\overline{B}_6$ of B_6 [7; p 34]. $\overline{B}_6$ is isotropic to B_6 and the invariant density under B_6 of the isotropic planes π, if it exists, coincides with the invariant density under $\overline{B}_6$ of the points *(u,v,w)* in the set of parameters [5; p 33]. The associated group $\overline{B}_6$ has the infinitesimal operators:

$$X_1 = -u\frac{\partial}{\partial w} \qquad X_2 = -v\frac{\partial}{\partial w} \qquad X_3 = -v\frac{\partial}{\partial u} \qquad X_4 = \frac{\partial}{\partial w}$$

$$X_5 = \frac{\partial}{\partial u} \qquad X_6 = -uv\frac{\partial}{\partial u} - (1 + v^2)\frac{\partial}{\partial v} - vw\frac{\partial}{\partial w}$$

The corresponding system of Deltheil is:

$$-u\frac{\partial f}{\partial w} = 0 \qquad -v\frac{\partial f}{\partial w} = 0 \qquad -v\frac{\partial f}{\partial u} = 0 \qquad \frac{\partial f}{\partial w} = 0$$

$$\frac{\partial f}{\partial u} = 0 \qquad -uv\frac{\partial f}{\partial u} - (1 + v^2)\frac{\partial f}{\partial v} - vw\frac{\partial f}{\partial w} - 4vf = 0$$

The system , has solution : $f(v)=\frac{c}{\left(1+v^2\right)^2}$, where c = const. The value $\frac{1}{\left(1+v^2\right)^2}$, has the following geometrical meaning : if ψ is the angle between the plane π and the plane *Oxy*, then $\frac{1}{\left(1+v^2\right)^2} = \cos^4\psi$.

Theorem 3.1 *The set of isotropic planes π: z = ux + vy + w is measurable with respect to the group B_6 and has the invariant density*

$$d\pi=\frac{du\wedge dv\wedge dw}{\left(1+v^2\right)^2}. \qquad (4)$$

If the plane π is expressed by parameters other than *u*, *v*, *w*, the density dπ will take different forms. Let the plane π be determined by three points A $(a,0,0) = \pi\cap Ox$, B $(0,b,0) = \pi\cap Oy$ and C $(0,0,c) = \pi\cap Oz$, $a \neq 0$, $b\neq 0$, $c\neq 0$. Then we have

$$u=-\frac{c}{a},\qquad v=-\frac{c}{b},\qquad w=c$$

and consequently

$$d\pi=\frac{b^2c^2}{a^2\left(b^2+c^2\right)^2}da\wedge db\wedge dc$$

Let $\overline{G} = \pi\cap Oxy$ be the horizontal trace of the plane π on the coordinate plane *Oxy*, and $\psi = \angle(\pi, Oxy)$. Then we have $\overline{G} = ux + vy + w$, $z = 0$ and $\cos\psi = \frac{1}{\sqrt{1+v^2}}$, where $\psi = \angle(\pi, Oxy)$. On the other hand, the plane *Oxy* is the Galilean plane, and the density $d\overline{G}$ of the straight lines $y=-\frac{u}{v}x+\frac{-w}{v}$ is [2; p. 44] $d\overline{G} = d\left(\frac{-u}{v}\right)\wedge d\left(\frac{-w}{v}\right)$.

For $d\psi$ and $d\overline{G}$ by differentiation we compute $d\psi = \cos^2\psi\, dv$ and $d\overline{G} = v^{-3}(v\, du\wedge dw - u\, dv\wedge dw - w\, du\wedge dv)$ .By exterior multiplication and comparing with (4) we get $dp = \sin^2\psi\, d\psi\wedge d\overline{G}$.

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